

Multiple quantifiers practice

Symbolize the following sentences using quantifiers.

1. Whoever owns a horse owns an animal.
2. Some dogs are smaller than all cats.
3. If Kasparov lost to a computer, then he was embarrassed.
4. If Kasparov lost to a computer, then the computer was designed by an IBM employee.
5. Some colleges are older than others, but no college is older than Harvard.
6. No football players are philosophy majors, but some are friends of philosophy majors.
7. Every state except Alaska has apple trees.
8. Every driver who owns a Humvee loves it.
9. Some Humvees are owned by basketball players.
10. If a basketball player owns a Humvee, then he is a millionaire.
11. If only basketball players own Humvees, then Humvees are expensive.
12. No driver who owns a Humvee is poor.
13. No driver who owns a Humvee sleeps in it.

Symbolize the following arguments using all the principles of first-order logic. Think about whether the arguments are valid or not, based on your understanding of the English arguments.

1. Any professional can outplay any amateur. Jones is a professional but he cannot outplay Meyers. Therefore Meyers is not an amateur.
2. If there are any policemen, then if there are any robbers, then they will arrest them. If any robbers are arrested by policemen, they will go to jail. There are some policemen and Macky is a robber. So Macky will go to jail.
3. Whoever is a friend of either Andrea or Bill will receive a gift. If Andrea has any friends, then Claire is one of them. If Daisy is a friend of Andrea, then Claire will receive a gift.

Determine whether the following arguments are valid. For invalid arguments provide an interpretation to show this.

1.
$$\begin{array}{|l} \forall x (Y(x) \vee B(x)) \\ \forall x (B(x) \rightarrow \neg G(x)) \\ \hline \forall x (G(x) \rightarrow Y(x)) \end{array}$$
2.
$$\begin{array}{|l} \forall x (D(x) \rightarrow M(x)) \\ \exists x \neg(D(x) \wedge M(x)) \\ \hline \exists x M(x) \end{array}$$
3.
$$\begin{array}{|l} \exists x (C(x) \wedge L(x)) \\ \forall x (L(x) \rightarrow F(x, a)) \\ \hline \exists x (C(x) \wedge F(x, a)) \end{array}$$
4.
$$\begin{array}{|l} \forall x [C(x) \rightarrow (S(x) \vee M(x))] \\ \forall x (M(x) \rightarrow B(x, b)) \\ \hline \forall x (B(x, b) \rightarrow C(x)) \end{array}$$