

Truth Tables and Semantic Concepts in TFL

Tautology

A sentence is a tautology if and only if it is true in every valuation

This is the case iff every row of the truth table assigns the value True to the sentence.

Determine whether the following sentences are tautologies.

1. $\neg A \vee \neg(B \wedge \neg A)$
2. $\neg(A \wedge B) \vee B$
3. $(\neg A \vee B) \vee \neg A$
4. $(A \wedge B) \vee ((B \vee C) \wedge \neg A)$
5. $(A \wedge \neg C) \vee (\neg(A \wedge B) \vee C)$

Contradiction – a sentence that is false in every valuation

Equivalence

A and B are equivalent iff for every valuation they have the same truth value, there is no valuation in which they have opposite truth values

Determine whether the following pairs of sentences are tautologically equivalent.

1. $\neg(\neg A \vee B) \vee (A \wedge B)$ and A
2. $A \vee \neg B$ and $\neg((\neg A \wedge B) \wedge \neg B)$
3. $A \wedge B$ and $(A \vee B) \wedge \neg A$
4. $A \wedge (\neg B \vee C)$ and $\neg(\neg A \vee C)$
5. $\neg A \vee B$ and $\neg(A \wedge C) \vee B$

Satisfiability (a surrogate for jointly possible)

A and B (or more) are jointly satisfiable if there is some valuation in which they are all true

Entailment (surrogate for validity)

The sentences A, B, etc, entail the sentence C if and only if there is no valuation that makes all of A, B, etc. true and C false.

(i.e. there is no counterexample row) the “double turnstyle” symbol represents entailment

For each of the five arguments below, determine whether the conclusion is entailed by the argument's premises.

1.
$$\begin{array}{|l} \hline A \vee B \\ \neg(A \wedge C) \\ \neg(B \wedge C) \\ \neg C \\ \hline \end{array}$$

3.
$$\begin{array}{|l} \hline B \\ \neg A \wedge (B \vee A) \\ \neg(\neg A \wedge B) \\ \hline \end{array}$$

5.
$$\begin{array}{|l} \hline \neg A \vee B \\ \neg(B \wedge \neg C) \\ \neg C \\ \hline \neg A \\ \hline \end{array}$$

2.
$$\begin{array}{|l} \hline \neg(\neg A \vee B) \\ \hline A \\ \hline \end{array}$$

4.
$$\begin{array}{|l} \hline A \\ \neg(C \wedge \neg B) \vee \neg B \\ \hline \end{array}$$