

Rules of Implication

1. *Modus ponens* (MP)

$$\begin{array}{l} \Box \supset \bigcirc \\ \underline{\Box} \\ \bigcirc \end{array}$$

3. *Hypothetical syllogism* (HS)

$$\begin{array}{l} \Box \supset \bigcirc \\ \bigcirc \supset \Delta \\ \hline \Box \supset \Delta \end{array}$$

5. *Constructive dilemma* (CD)

$$\begin{array}{l} (\Box \supset \bigcirc) \cdot (\Delta \supset \Diamond) \\ \underline{\Box \vee \Delta} \\ \bigcirc \vee \Diamond \end{array}$$

7. *Conjunction* (Conj)

$$\begin{array}{l} \Box \\ \underline{\bigcirc} \\ \Box \cdot \bigcirc \end{array}$$

2. *Modus tollens* (MT)

$$\begin{array}{l} \Box \supset \bigcirc \\ \underline{\sim \bigcirc} \\ \sim \Box \end{array}$$

4. *Disjunctive syllogism* (DS)

$$\begin{array}{l} \Box \vee \bigcirc \\ \underline{\sim \Box} \\ \bigcirc \end{array}$$

6. *Simplification* (Simp)

$$\begin{array}{l} \underline{\Box \cdot \bigcirc} \\ \Box \end{array}$$

8. *Addition* (Add)

$$\begin{array}{l} \underline{\Box} \\ \Box \vee \bigcirc \end{array}$$

Rules of Replacement

9. *De Morgan's rule* (DM)

$$\begin{array}{l} \sim(\Box \cdot \bigcirc) :: (\sim \Box \vee \sim \bigcirc) \\ \sim(\Box \vee \bigcirc) :: (\sim \Box \cdot \sim \bigcirc) \end{array}$$

10. *Commutativity* (Com)

$$\begin{array}{l} (\Box \vee \bigcirc) :: (\bigcirc \vee \Box) \\ (\Box \cdot \bigcirc) :: (\bigcirc \cdot \Box) \end{array}$$

11. *Associativity* (Asso)

$$\begin{array}{l} [\Box \vee (\bigcirc \vee \Delta)] :: [(\Box \vee \bigcirc) \vee \Delta] \\ [\Box \cdot (\bigcirc \cdot \Delta)] :: [(\Box \cdot \bigcirc) \cdot \Delta] \end{array}$$

12. *Distribution* (Dist)

$$\begin{array}{l} [\Box \cdot (\bigcirc \vee \Delta)] :: [(\Box \cdot \bigcirc) \vee (\Box \cdot \Delta)] \\ [\Box \vee (\bigcirc \cdot \Delta)] :: [(\Box \vee \bigcirc) \cdot (\Box \vee \Delta)] \end{array}$$

13. *Double negation* (DN)

$$\Box :: \sim \sim \Box$$

14. *Transposition* (Trans)

$$(\Box \supset \bigcirc) :: (\sim \bigcirc \supset \sim \Box)$$

15. *Material implication* (Imp)

$$(\Box \supset \bigcirc) :: (\sim \Box \vee \bigcirc)$$

16. *Material equivalence* (Equiv)

$$\begin{array}{l} (\Box \equiv \bigcirc) :: [(\Box \supset \bigcirc) \cdot (\bigcirc \supset \Box)] \\ (\Box \equiv \bigcirc) :: [(\Box \cdot \bigcirc) \vee (\sim \Box \cdot \sim \bigcirc)] \end{array}$$

17. *Exportation* (Exp)

$$[(\Box \cdot \bigcirc) \supset \Delta] :: [\Box \supset (\bigcirc \supset \Delta)]$$

18. *Tautology* (Taut)

$$\begin{array}{l} \Box :: (\Box \vee \Box) \\ \Box :: (\Box \cdot \Box) \end{array}$$

Rules of Supposition

Conditional Proof (CP)

1. $\Box$
 2. $\bigcirc$ / $\Delta \supset \Diamond$
 | 3. Δ ACP
 | $\vdots$
 | $n.$ $\Diamond$
 $n+1.$ $\Delta \supset \Diamond$ 3– n , CP

Indirect Proof (IP)

1. $\Box$
 2. $\bigcirc$ / Δ
 | 3. $\sim \Delta$ AIP
 | $\vdots$
 | $n.$ $\Diamond \cdot \sim \Diamond$
 $n+1.$ $\sim \sim \Delta$ 3– n , IP
 $n+2.$ Δ n+1, DN